Fuzzy logic control for flexible link robot arm by singular perturbation approach

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Abstract

This work addresses flexible-link robot arm control using fuzzy logic, and a singular perturbation approach. A singular perturbation approach is introduced to derive the slow and fast subsystems and thus reduce the effect of spillover. Consequently, a two-time scale fuzzy logic controller will be applied for such systems. The fast-subsystem controller damps out the vibration of the flexible structure by the optimal control method. Thus, the slow-subsystem fuzzy controller dominates the tracking of the trajectory. The stability of the internal dynamics was guaranteed by adding a boundary-layer correction based on singular perturbations. The first investigation of hybrid LQ-fuzzy controller yielded a favorable output response for the flexible link robot arm than does the traditional outer-loop PD control with the boundary layer stabilizer. The fuzzy control method is reliable and robust.

Keywords: Flexible-link; Singular perturbation; Fuzzy logic control; Composite control

1. Introduction

A flexible link arm is a distributed parameter system of infinite order, but must be approximated by a lower-order model and controlled by a finite-order controller due to onboard computer limitations, sensor inaccuracy, and system noise. The so-called “control spillover” and “observation spillover” effects then occur, which under certain conditions can lead to instability [3,12,13].

A controller that is based upon a reduced-order model is proposed in [6,9] to maintain reasonable computational loading. In recent years, singular perturbation theory has been demonstrated to provide a convenient means “reduced-order modeling”. The dynamics of singularly perturbed systems can be approximated by the dynamics of the corresponding reduced-order and boundary layer subsystems for sufficiently small values of the singular perturbation parameter. The aim is to simplify the software and hardware implementation of control algorithms, while improving their robustness. A composite control approach, based on a two-time scale model of the flexible-link arm has been derived in [15,19], and allows a definition of a slow subsystem that corresponds to a rigid body and a fast subsystem that describes the flexible motion. The approach also supports a fast subsystem that describes the flexible motion.
For many years, classical control engineers began their work with a mathematical model and did not acquire further knowledge of the system. Today, control engineers use all of the above sources of information. Aside from a mathematical model whose use is clear, numerical (input/output) data can be considered to develop an approximate model and a controller, based on fuzzy IF-THEN rules [24,26].

The application of the concepts of fuzzy set theory in structural control has recently attracted increasing interest. Fuzzy controllers afford a simple and robust framework for specifying nonlinear control laws that accommodate uncertainty and imprecision. Such controllers may be implemented using a fuzzy mathematical model of the plant and controller. Fuzzy controllers may be determined without a mathematical model when linguistic descriptions of the control are available or can be formulated. Implementations that depend on a linguistic synthesis approach have been proposed [7,8,17,23–25,27] and shown to be theoretically and practically applicable.

This work examines the feasibility of applying control algorithms based on fuzzy logic to a class of hybrid structural control systems shown in [1,10,16,18]. The investigation included both analytical and experimental verification of the fuzzy control algorithm. The guidelines for implementing a fuzzy active control strategy for civil engineering structures are discussed in [5,20,21]. Those studies focus on the gap between a successful numerical example and the technical design of the device.

This work concentrates mainly on fuzzy logic, and the use of the singular perturbation approach in flexible-link robot arm control. A singular perturbation approach is also introduced to derive the slow and fast subsystems and thus reduce the effect of spillover. Therefore, a two-time scale fuzzy logic controller will be applied for such systems. The fast-subsystem controller will dampen the vibration of the flexible structure by the optimal control method. Hence, the slow-subsystem fuzzy controller dominates the tracking of the trajectory. The stability of the internal dynamics is guaranteed by adding a boundary-layer correction, based on singular perturbations. This fuzzy logic/singular perturbation approach brings together and employs the concepts elucidated in several of the papers mentioned above.

2. Robot dynamics and sensor system design

2.1. Robot dynamics

A convenient and approximate form of the dynamics of a flexible robot arm can be derived by the assumed mode shape method. The deflection of elastic beam $w(\eta, t)$ can be expressed as a summation of the infinite series terms by assuming a Bernoulli–Euler beam model [14]:

\[ w(\eta, t) = \sum_{i=1}^{\infty} q_i(t)\phi_i(\eta), \]

where $q_i(t)$ are generalized modal coordinates and $\phi_i(\eta)$ are mode shape functions that depend on the boundary-value problem (i.e. pinned-pinned, clamped-free, clamped-loaded, ...), where $\eta$ represents displacement along the neutral axis of the link.

The components of the dynamic model should be explicitly separated into matrix form to specify the inertia ($M$), centrifugal/Coriolis/damping ($D$), stiffness ($K$), friction $F(x, \dot{x})$, and gravitational constant $G$.

\[ M(x)\ddot{x} + D(x, \dot{x})\dot{x} + K(x)x + F(x, \dot{x}) + G(x) = B\tau, \]

where $B$ is the input matrix that depends on the clamped link assumptions, and $\tau$ represents the control input torque. Here, $\dot{x} = [q_0, q_1, \ldots, q_n]^T$ is defined, where $q_0$ is the rigid mode, $q_1, \ldots, q_n$ are the flexible modes, and $n$ is the number of retained modes in Eq. (1).

A pseudo-linear state-space representation for the flexible arm is given by

\[ X = \begin{bmatrix} 0 & I \\ -M^{-1}K -M^{-1}D \\ M^{-1}B \end{bmatrix} X + \begin{bmatrix} 0 \\ -M^{-1} \end{bmatrix} \tau_n, \]

with $X = [x, \dot{x}, \ddot{x}]^T$, and the nonlinear forcing terms, $\tau_n = [F(x, \dot{x}) + G(x)]$.

Given the robot dynamics (3), the output is defined as

\[ y = Cq. \]
where \( C \) is generally a function of \( q \). The aim here is to determine the control torque inputs \( \tau(t) \) that guarantee suitable performance with respect to the behavior of \( y(t) \).

### 2.2. Sensor system design

#### 2.2.1. Rigid mode measurement

Each axis of rotation uses an optical increment encoder that is mounted on the motor to measure position. The measurement of position is extracted from the quadrature signals by incrementing or decrementing a counter, as appropriate with every pulse edge.

Eq. (5) specifies the measurement of the joint angle \( \phi = [\phi_1'(0), \phi_2'(0), \ldots, \phi_n'(0)]^T \).

\[
[\psi] = \begin{bmatrix} 1 & \phi_1'(0) & \phi_2'(0) & \cdots & \phi_n'(0) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}
\]

#### 2.3. Flexible mode measurement

Several sensors have been used to measure the vibration of a flexible beam. The most popular has been the strain gauge, which measures the deflection of the beam. The advantages of strain gauges are isolation of beam variables from rigid rotations, lack of restrictions on work positioning, high compatibility with harsh industrial environments, and low cost. The relationship between the strain, \( \delta \), and the generalized coordinates \( q_i(t) \) is

\[
\delta = c \sum_{i=1}^{\infty} \left[ \begin{array}{c} \phi_1''(a) \\ \phi_2''(a) \\ \cdot \cdot \cdot \\ \phi_n''(a) \end{array} \right] d \phi_i(a) dt^2
\]

where \( c \) is the curvature of the beam.

Eq. (6) can be expanded to relate each flexible mode to the measured strain at each location, \( a, b, \ldots, m \). This relationship can be presented in matrix form as

\[
\begin{bmatrix} \delta(a, t) \\ \delta(b, t) \\ \vdots \\ \delta(m, t) \end{bmatrix} = [c] \begin{bmatrix} \frac{d^2\phi_1(a)}{dx^2} \\ \frac{d^2\phi_2(a)}{dx^2} \\ \cdot \cdot \cdot \\ \frac{d^2\phi_n(a)}{dx^2} \end{bmatrix}
\]

### 3. Link-tip position control using I/O feedback linearization

This section applies input/output (I/O) feedback linearization to design a controller for the link-tip positions, \( y \in \mathbb{R}^n \). Moreover, Section 4 completes the design using singular perturbation theory to design a controller that gives a boundary-layer correction to stabilize the flexible modes.

#### 3.1. Input/output feedback linearization

In Eq. (4) \( y(t) \) is twice differentiated using standard I/O feedback linearization techniques, and is then substituted for \( \ddot{q} \), using Eq. (2) to obtain the reduced-order system,

\[
\ddot{y} = u.
\]

The auxiliary control input, \( u(t) \), is defined by the reduced-order computed torque (ROCT) control
law,
\[ \tau = (CM^{-1}B)^{-1}(CM^{-1}(Dq + Kq + F + G) + u). \]
(10)

System (9) consists of \( n \) subsystems, each with two integrators in series, and is said to be in Brunovsky canonical form.

When a finite number of flexible modes are retained in the dynamic model, the matrix \( CM^{-1}B \) is nonsingular. This matrix has been shown to become singular as the number of modes approaches infinity as in the exact model. In fact, retaining a finite number of modes corresponds to "approximate feedback linearization" of systems with an ill-defined relative degree, and is sufficient in designing controllers with adequate performance.

The tracking error, \( e(t) = y_d(t) - y(t) \), is defined with \( y_d(t) \) as the desired trajectory, and \( K_d > 0 \), \( K_r > 0 \) is chosen. Then, an outer-loop PD control law for a trajectory is
\[ u = \ddot{y}_d + K_d \ddot{y} + K_r e. \]
(11)

Unfortunately, this obvious selection for \( u(t) \) dooms the control scheme. Even though the ROCT control consists of (11), and (10) decouples the \( e(t) \) subsystem from the remainder of the plant and stabilizes it, the ROCT almost certainly fails to stabilize the remainder of the plant because the zero dynamics are unstable. This issue was explored in [19,22].

3.2 Inverse system dynamics

Partitioned matrices were used as follows to elucidate the detailed structure of the closed-loop system after ROCT. Eq. (2) is rewritten, neglecting friction for simplicity as

\[ \begin{bmatrix} M_a & M_b \\ M_b & M_b \end{bmatrix} \begin{bmatrix} \dot{q}_f \\ \dot{q}_r \end{bmatrix} + \begin{bmatrix} D_a & D_b \\ D_b & D_b \end{bmatrix} \begin{bmatrix} q_f \\ q_r \end{bmatrix} = \begin{bmatrix} B_f \\ B_r \end{bmatrix} \tau. \]
(12)

The inverse of the mass matrix can be defined as
\[ \begin{bmatrix} M_a & M_b \\ M_b & M_b \end{bmatrix}^{-1} = \begin{bmatrix} M_a \\ M_b \end{bmatrix}^{-1}. \]
(13)

and Eq. (12) can be multiplied by Eq. (13) from the left, and the terms rearranged:
\[ \begin{align*}
\dot{q}_f &= -D_a^Tq_f - D_b^Tq_r - K_a^Tq_f - G_a^T \tau \\
\dot{q}_r &= -D_b^Tq_f - D_b^Tq_r - K_r^Tq_r - G_r^T \tau
\end{align*} \]
(14)

with
\[ D_a^T = H_aD_a + H_dD_q, \quad D_b^T = H_bD_q + H_dD_h, \]
\[ D_a^T = H_aD_a + H_dD_q, \quad D_b^T = H_bD_q + H_dD_h, \]
\[ K_a^T = H_aK_{aH}, \quad K_r^T = H_dK_{rH}, \]
\[ G_a^T = H_aG_a, \quad G_r^T = H_dG_r, \]
\[ B_f^T = H_aB_f + H_dB_r, \quad B_r^T = H_dB_f + H_qB_r. \]

In the case of the general flexible-link robot arm, \( B_f \) is smaller than \( B_r \). A sufficient condition that guarantees the existence of \((B_r^T)^{-1}\) in terms of the matrix-induced norm (for example, the maximum singular value [11]) is easily shown to be
\[ ||B_f|| < ||(B_r^T)^{-1}H_d||^{-1}. \]
(15)

Performing feedback linearization to (14) is equivalent to selecting
\[ \tau = (B_r^T)^{-1}(D_a^Tq_f + D_b^Tq_r + K_a^Tq_f + G_a^T + u) \]
(16)
to obtain the dynamics
\[ \dot{q}_f = u \]
(17)
\[ \dot{q}_r = -D_b^Tq_f - D_b^Tq_r - K_r^Tq_r - G_r^T + B_r^Tu, \]
(18)

where \( u(t) \) is an auxiliary input and the Schur complements are defined as
\[ D_a^T = D_a^T - B_a^T(R_r)^{-1}D_a^T, \]
\[ D_b^T = D_b^T - B_a^T(R_r)^{-1}D_b^T, \]
\[ K_a^T = K_a^T - B_a^T(R_r)^{-1}K_a^T, \]
\[ G_a^T = G_a^T - B_a^T(R_r)^{-1}G_a^T, \]
\[ B_f^T = B_f^T(R_r)^{-1}. \]
A state-space representation of the dynamics is

\[
\begin{bmatrix}
\dot{q}_r \\
\dot{q}_f \\
\dot{q}_s \\
\ddot{q}_f
\end{bmatrix} =
\begin{bmatrix}
0 & I & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & I \\
0 & -P^b_{rr} & -K^b_{ff} & -B^b_{ff}
\end{bmatrix}
\begin{bmatrix}
q_r \\
q_f \\
q_s \\
\dot{q}_f
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
I
\end{bmatrix} u + \begin{bmatrix}
0 \\
0 \\
0 \\
-I
\end{bmatrix} G^b_f (19)
\]

Control law (16) is a refined expression of the ROCT control (10), and system (19) is the complete closed-loop system after ROCT. The given rigid-mode trajectory, \( y_d(t) \), in (17) can be obtained by appropriate selection of \( u(t) \). Consequently, the function of the inverse system is to construct the unique flexible mode trajectory that is associated with the corresponding rigid-mode trajectory. Selecting \( u(t) \) based only on the desired rigid subsystem performance does not guarantee a stable inverse system, because the zero dynamics are not stable for the flexible-link robot arm. Hence, the following section selection of \( u(t) \) to achieve the desired rigid-mode performance and stabilize the inverse dynamics, using a time scale separation in the next section.

### 4. Stabilizing internal dynamics using boundary-layer correction

This section applies singular perturbation theory to stabilize the zero dynamics after I/O feedback linearization. As will be seen, this approach affords greater accuracy of the time-scale separation assumption. A typical procedure for singular perturbation theory is reviewed in [9]. The feedback-linearized system is singularly perturbed by rewriting (17) and (18) as

\[
\dot{q}_r = u,
\]

\[
\dot{q}_f + D^b_{rr} \dot{q}_r + D^b_{ff} \dot{q}_f + K^b_{ff} \dot{q}_f + G^b_f = B^b_{ff} u. (21)
\]

The singular perturbation analysis depends on putting (20) and (21) in standard form and solving for some fast variables. An appropriate parameter must first be extracted from the fast subsystem because the flexible subsystem is faster than the rigid one.

By defining

\[
U = H_d - \{[H_u B_f + H_d B_i]^{-1} H_d \} \times (H_u B_f + H_d B_i)^{-1} H_d
\]

(22)

\[
K^b_d = K^b = UK_d
\]

(23)

where \( K^b \) indicates "closed-loop stiffness" and \( K_d \) denotes the open-loop stiffness that appears in Eq. (16). \( M^b \) is generally large, such that \( K^b_d = UK_d \) is of a larger scale than \( K_d \). Therefore, introducing a singular perturbation based on \( K^b_d \) rather than on \( K_d \) is more accurate. Therefore, a positive scaling factor, \( \kappa \), and factor, \( K^b \), are introduced as

\[
K^b = \kappa K^b_d
\]

(24)

where \( K^b_d \) is an invertible constant matrix and is independent of the closed-loop stiffness factor, \( \kappa \).

Variables \( \dot{e}^2 = (1/\kappa) \) and \( \dot{e}^2 \dot{\xi}(t) = y(t) \) are then defined. This work addresses a case in which the stiffness \( \kappa \) is sufficiently large. The primary aim of the control design is to enable the rigid motion, \( y(t) \), to track a desired trajectory, \( y(t) \). The output, \( y(t) \), can be generally written as \( y = q_r + f(q,t) \). In the pinned-pinned mode case, \( f(t) = 0 \), while in the clamped-free mode case, \( f(t) \) is not constantly zero. Accordingly, the output of the system is set as, \( y = y_d(t) \); restated a modified output is used in the clamped-free case. Therefore, rewriting (21), (22) in terms of \( \dot{\xi}(t) \) and \( \dot{\xi} \), yields

\[
\dot{y}(t) = u,
\]

\[
\dot{\xi} = -D_{rr} \dot{q}_r \dot{\xi} + D_{ff} \dot{q}_f \dot{\xi} + G_f \dot{\xi} + B_{ff} u.
\]

(25)

For simplicity, the superscript "b" has been dropped. A control, \( u \), is designed to allow \((\gamma, \gamma) \) track \((\gamma_d, \gamma_d) \) closely and to stabilize the systems (25) and (26).

The control input

\[
u = \ddot{u} + u
\]

(27)

is defined as \( \ddot{u} \) as the slow control component and \( u \) as a fast control component. Setting \( \dot{e} = 0 \) yields the slow dynamics

\[
\dot{\gamma} = \dot{u},
\]

(28)
and an algebraic relationship in \( \hat{y}, \hat{\xi}, \) and \( \hat{\xi} \).

\[
0 = -\dot{\mathbf{D}}_y \hat{y} - \mathbf{K}^s \hat{\xi} - \dot{\mathbf{G}}_t + \mathbf{B}_t \ddot{u}.
\]  

(29)

Notably, an overbar denotes the evaluation of nonlinear functions, with \( \varepsilon = 0 \).

Expressing \( \varepsilon = 0 \) in Eq. (29) yields the slow manifold equation:

\[
\dot{\xi} = (\mathbf{K}^s)^{-1} (-\dot{\mathbf{D}}_y \hat{y} - \dot{\mathbf{G}}_t + \dot{\mathbf{B}}_t \ddot{u})
\]  

(30)

The dynamics (28) are valid on this manifold. States \( \xi_1 \equiv \xi - \hat{\xi} \) and \( \xi_2 \equiv \varepsilon \hat{\xi} \) are selected and Eq. (29) rewritten as:

\[
\varepsilon \dot{\xi}_2 = -\mathbf{D}_y \dot{\hat{y}} - \mathbf{D}_y \hat{\xi}_2 - \mathbf{K}^s (\xi_1 + \hat{\xi}) - \mathbf{G}_t + \mathbf{B}_t \ddot{u},
\]  

(31)

where a time-scale change of \( \tau = t/\varepsilon \) yields:

\[
\frac{d\xi_1}{d\tau} = \xi_2
\]

\[
\frac{d\xi_2}{d\tau} = -\mathbf{D}_y \dot{\hat{y}} - \mathbf{D}_y \xi_2 - \mathbf{K}^s (\xi_1 + \hat{\xi}) - \mathbf{G}_t + \mathbf{B}_t \ddot{u}
\]  

(32)

Setting now \( \varepsilon = 0 \) and substituting for \( \dot{\xi} \) from Eq. (30) gives the fast dynamics part of the following slow subsystem description of the feedback-linearized arm:

\[
\dot{\hat{y}} = \bar{u}
\]  

(33)

Moreover, the fast subsystem can be found to be

\[
\frac{d\xi_1}{d\tau} = \xi_2
\]

\[
\frac{d\xi_2}{d\tau} = -\mathbf{K}^s \xi_1 + \mathbf{B}_t u(t)
\]  

(35)

which describes a linear system parameterized in the slow variables.

4.1 Tracking requirement

According to (33)-(35), \( u(t) \) can be selected to yield stable zero dynamics, provided that \((\mathbf{K}^s, \mathbf{B}_t)\) is stabilizable. If \((\mathbf{K}^s, \mathbf{B}_t)\) is controllable, no zero dynamics exist, and \( u(t) \) can be selected to control \((\xi_1^s, \xi_2^s)^T \).

A composite control strategy can be pursued, as demonstrated by the two reduced-order subsystems (33)-(35). The design of a feedback control for the full systems (25) and (26), can be split into two separate designs of feedback controls, \( \tilde{u}(t) \) and \( u(t) \), for the two reduced-order systems. Formally,

\[
u = \tilde{u} + u(t).
\]  

(36)

Moreover, a modified tracking output \( y \) is considered in [22]. Using Tikhonov’s Theorem,

\[
y = \hat{y} + O(\varepsilon),
\]  

(37)

\[
q_1 = \varepsilon^2 \xi_2 + \zeta_1 + O(\varepsilon),
\]  

(38)

where \((\hat{\xi}(t))\) is given by Eq. (30) and \( O(\varepsilon) \) denotes terms of order \( \varepsilon \). \( \tilde{u}(t) \) is thus selected for suitable tracking behavior in the slow subsystem (33). Then, \( u(t) \) is selected for the stability of the fast part (34) and (35). The fast control law is used to stabilize the inverse system dynamics since the fast subsystem is nothing but a linearization of the flexible (internal system) dynamics induced by the slow manifold \( \hat{\xi}(t) \). In this work, \( u(t) \) can be designed as an optimal control for the boundary layer. The performance index will be a function of the slow state variables. The feedback gain matrices can also be designed on the basis of the final joint configuration, if and only if for that particular choice of \( \zeta_1, \zeta_2 \) will not be unstable on the slow trajectory, since the main purpose of a flexible link robot arm control is to dampen the steady state deflections as rapidly as possible. Accordingly, the need to solve a Riccati equation for each joint configuration can be avoided [19].

Substituting the composite control (36) into the control law (16) gives the entire control, applied to the flexible-link robot arm.

In this work, Eq. (38) shows that the flexible mode trajectory has two parts. The slow part, \( \hat{\xi}(t) \) in (30), depends on the rigid mode trajectories and the control \( \hat{u}(t) \) used to construct them. The fast part, \( \xi_1 \), is a boundary-layer correction added for stabilization.

The composite control requires full state feedback of the rigid and flexible modes and their derivatives.
In practice, an observer might be required to reconstruct the velocities from position and strain gauge measurements. Such techniques are commonly described in the literature [6,15] and straightforwardly applicable.

5. Fuzzy control

The field of fuzzy system and control has seen great progress, motivated by practical success in controlling industrial processes. Fuzzy systems can be used as closed-loop controllers. In such cases, a fuzzy system measures the outputs of the process and continuously controls the actions involved in the process. The fuzzy controller uses a form of quantification of imprecise information (input fuzzy sets) to generate by an inference scheme, which is based on a knowledge base of control force to be applied to the system [20].

The benefit of this quantification is that the fuzzy sets can be represented by a unique linguistic expression, such as small, medium, or large. The linguistic representation of a fuzzy set is known as a term, and a collection of such terms defines a term-set, or a library of fuzzy sets. Fuzzy control converts a linguistic control strategy, typically based on expert knowledge, into an automatic control strategy. The logical controller is comprised of four primary components. (1) Fuzzifier (the values of the state variables monitored during the process are fuzzified into fuzzy linguistic terms); (2) a knowledge base that contains fuzzy IF-THEN rules and membership functions; (3) fuzzy reasoning (the result of which is a fuzzy output for each rule), and (4) a defuzzification interface (which produces a crisp control output) [5].

Fuzzy processing enables the quantification of an irregular input by sets that can be treated according to their significance. A rule in the knowledge base can be defined, such that the control force will be zero, without detrimentally affecting the system. Similarly, any set can be treated differently from any other set. The fuzzy controller offers several advantages. (1) It can discriminate among groups of input values of high or low importance, and treat them accordingly, with a small number of operations; (2) it can easily eliminate actions that may be detrimental to the system; (3) using trapezoidal fuzzy sets implicitly filters input signals, and (4) output aggregation and defuzzification schemes reduce the influence of input noise and computer imprecision.

5.1. Fuzzy control structure of the system

5.1.1. Fast subsystem stabilizer $u_f$

The control scheme of the flexible link robot arm must be able to control the motion in the rigid body mode and suppress the vibration modes of the arm. In this work, the fast subsystem controller is designed to stabilize all the vibration modes of the arm. Therefore, an LQ design was applied to the fast subsystem to select appropriate PD gains.

5.1.2. Slow subsystem fuzzy controller $\bar{u}$

A slow subsystem fuzzy controller is designed, using the tracking error, $e$, and error rate, $\dot{e}$, to implement input/output feedback linearization/singular perturbation approach control. Hence, $\bar{u}$ is selected for suitable tracking in the slow subsystem. Fig. 6 shows block diagrams of two-time scale fuzzy logic controller for the flexible link robot arm.

5.2. Membership function and fuzzy rule base

Two difficulties in designing any fuzzy control system are the shape of the membership functions and the choice of the fuzzy rules. The decision-making logic is the way in which the controller output is generated. The decision-making uses input fuzzy sets, and the decision is governed by the values of the inputs. Furthermore, the knowledge base consists of knowledge of the application domain and the attendant control goals. It includes a database and a fuzzy control rule base.

A fuzzy logic controller is designed using the tracking error, $e$, and error rate, $\dot{e}$, to implement input-output feedback linearization/singular perturbation approach control. Very many researchers have recommended the use of equally spaced triangular membership functions, “common sense” rules, and then finding a way to “modify”, “tune”, or “adapt” them to the plant’s variations, unmodelled dynamics, or external disturbances. A control system is said to be an adaptive fuzzy control system if a set of fuzzy rules is used either to modify or to change an existing fuzzy controller’s architecture (including
Fig. 1. Configuration of flexible-link robot arm.

Fig. 2. Membership function of $e$.

Fig. 3. Configuration of flexible-link robot arm.

Fig. 4. Membership function of $\dot{e}$.

membership functions and/or rules). Figs. 1–3 show fuzzy membership functions of $e$ and its derivative, $\dot{e}$. Seven fuzzy membership functions correspond to large negative (LN), medium negative (MN), small negative (SN), zero (ZE), small positive (SP), medium positive (MP), large positive (LP) values of the error, $e$, and the error rate, $\dot{e}$. The support vectors, $e$ and $\dot{e}$, are between $-0.1$ and 0.1. However, neither $e$ nor $\dot{e}$ are restricted to belong to this support, since the fuzzy membership function can be extended to $\pm \infty$. Therefore, any $e$ or $\dot{e}$ that exceeds 0.1 will be involved the large positive (LP) membership function.

Fig. 4 presents the fuzzy set of the slow subsystem controller output, $\bar{u}$. Seven fuzzy membership functions are the same as $e$ and $\dot{e}$.

After the input and output values are assigned to define fuzzy sets, each possible input condition must
be mapped onto an output condition. Such mapping is commonly expressed as follows. 

If (condition or antecedent) then (action or consequence): the fuzzy rule base can be illustrated as a look-up table. Table 1 presents a look-up table that corresponds to the fuzzy sets depicted in Figs. 5 and 6. This look-up table is a matrix of seven rows (the number of membership functions of the \( e \) fuzzy-set)
5.3. Defuzzification

Defuzzification is the conversion of a fuzzy quantity represented by a membership function, into a precise or crisp value. Commonly used strategies may be described as maximum criterion, mean of maximum, and centroid methods. This study uses the centroid and the singleton methods to combine and defuzzify outputs into crisp values. The centroid method is first used to determine each fired output value. A singleton or average weight method is used to combine those output values into an executable single value. The advantage of the center of gravity defuzzifier is its intuitive plausibility, but its disadvantage is its computational intensity.

6. Results and discussion

(1) \( \varepsilon = 0.05 \). Fig. 7 presents the tip position tracking error of the flexible link robot arm with outer-loop PD control and boundary-layer. Fig. 8 shows the tip position tracking error of the flexible link robot arm, with fuzzy logic control and a boundary-layer stabilizer. \( y(t)_{\text{fuzzy}} \) clearly tracks the desired trajectory, \( y(t)_{\text{desired}} \), very closely. Moreover, Figs. 9 and 10 reveal that the flexible mode deflections, using the fuzzy logic controller, are damped out more quickly than the outer-loop PD controller. In the case of fuzzy logic control, the elastic vibrations virtually vanish after a short period and remain at zero.

(2) \( \varepsilon = 0.012 \). Fig. 11 shows the response of the tip position and the tip position tracking when \( \varepsilon = 0.012 \). Figs. 12–14 depicts the response of the position-tracking error in using fuzzy logic controller. The figure shows that the fuzzy logic
control used here is better than that which uses outer-loop PD control in tip position tracking and in flexible modes.

(3) 

**Varied ε.** Next, the performance of this composite controller is shown to improve when a more appropriate value of ε is used. Comparing the performance of the system using ε = 0.05 with that using ε = 0.012, shows that the vibration is damped out more quickly when ε is smaller.

However, using a fuzzy logic controller to obtain vibration responses does not differ from. The fuzzy control method appears to be quite reliable and robust. Fuzzy logic controllers are very robust controllers in nonlinear systems because they act only on rules that are applied to measured outputs, and can thus handle variations that a standard controller may not consider, for lack of explicit methods.
Fig. 8. Tip position tracking error of flexible link robot arm with fuzzy logic control and boundary-layer stabilizer ($\varepsilon = 0.05$).

Fig. 9. Response of flexible modes of flexible link robot arm with outer-loop control and boundary-layer stabilizer ($\varepsilon = 0.05$).

Fig. 10. Response of flexible modes of flexible link robot arm with fuzzy logic control and boundary-layer stabilizer ($\varepsilon = 0.05$).
Fig. 11. Tip position tracking error of flexible link robot arm with outer-loop PD control and boundary-layer stabilizer ($\varepsilon = 0.012$).

Fig. 12. Tip position tracking error of flexible link robot arm with fuzzy logic control and boundary-layer stabilizer ($\varepsilon = 0.012$).

Fig. 13. Response of flexible modes of flexible link robot arm with outer-loop PD control and boundary-layer stabilizer ($\varepsilon = 0.012$).
7. Conclusion

The primary contribution of this paper is in fuzzy logic, by providing a singular perturbation approach for flexible-link robot arm control. A singular perturbation approach was introduced to reduce the effect of spillover and thus derive slow and fast subsystems. A composite control design was adopted. Consequently, a two-time scale fuzzy logic controller can be applied for such systems. The fast-subsystem controller dampened out the vibration of the flexible structure by the optimal control method. Hence, the slow-subsystem fuzzy controller dominated the trajectory tracking. The stability of the internal dynamics was guaranteed by adding a boundary-layer correction based on singular perturbations. In conclusion, the first investigation of hybrid LQ-fuzzy controller yielded a favorable output response for the flexible link robot arm than does the traditional outer-loop PD control with the boundary layer stabilizer. The fuzzy controllers are potential candidates for deriving control in the presence of these structural nonlinearities. Future work will emphasize a two-time scale hierarchical fuzzy logic controller of the multilink flexible arm and will be appear in a forthcoming issue.

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