Electricity Load Forecast using Functional Networks

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Abstract. In this paper a model using a functional network was employed to approach the problem of electricity load forecasting. Functional networks are generalised neural networks, that permit the specification of their initial topology using knowledge about the problem at hand. In this case, and after analysing the available data and their relations, an additive model was chosen, from which different alternatives were attempted until we developed three different models: one for the prediction of load for non-week-end days, another one for Saturdays and a third one for Sundays and weekends. The MAPE errors obtained were in the interval [1.857, 1.973] during training, and in the interval [3.430, 6.396] during testing.

1 Introduction

This paper describes the application of functional networks [1,2] to a regression problem. The work has been carried out in the context of the World-wide competition within the EUNITE network - the European Network of Excellence on Intelligent Technologies for Smart Adaptive Systems. This European Network was started on January 2001 with the aim to exploit the synergy among the various Intelligent Technologies —neural networks, fuzzy systems, methods from machine learning, and evolutionary computing— in order to build Smart Adaptive Systems, and it is funded by the Information Society Technologies Programme (IST) within the European Union's Fifth RTD Framework Programme .

The competition is announced as the application of Intelligent and Adaptive technologies in electricity load forecast and it is linked to the Eastern Slovakian Electricity Corporation. The aim is to develop a model capable of predicting the maximum daily electrical load on the basis of the past mean temperature and load demands. This analysis is motivated by the significant financial profit that it is expected to bring when using more accurate prediction technology that the actual available. For this purpose, the competitors are given the following data:

a) Average daily temperatures of the time period 1995 – 1998
b) Half an hour daily loads of the time period 1997 – 1998
c) The holidays for the period 1997-1999

The problem is to supply a prediction for the 31 maximum daily values of electrical loads corresponding to the month of January of 1999.

To evaluate the quality of the results two measures will be used. First, the magnitude of the *MAPE error* defined as:

$$MAPE = 100 \cdot \frac{\sum_{i=1}^{n} \left| \frac{L_{R_i} - L_{P_i}}{L_{R_i}} \right|}{n} \quad . \qquad \textbf{(1)}$$

where n is the number of data predicted, L_{R_i} is the real value of maximum daily electrical load on the *i-th* day, i.e., the goal of the prediction, and L_{P_i} is the predicted value of maximum daily electrical load for that day.

The second evaluation magnitude is the *maximal error* which is defined as:

$$M = \max(\left| L_{R_i} - L_{P_i} \right|) \quad . \qquad \textbf{(2)}$$

where i is the number of predictions.

The problem has been approached from the point of view of functional networks. These are powerful models that have been employed to solve several practical problems, including regression problems, showing a better performance than typical neural networks [3].

In next section we describe functional networks in more detailed. Later on we will analyse the particular model developed for this competition, as well as the results obtained.

2 Functional Networks

Functional networks are a generalisation of neural networks, which is achieved by using multiargument and learnable functions [1,2], i.e., in this models transfer functions associated with neurons are not fixed but learned form data. Neither there is a need to include weights to ponder links among neurons since their effect is subsumed by the neural functions. **Fig. 1** shows an example of a general functional network for k=2 explanatory variables.

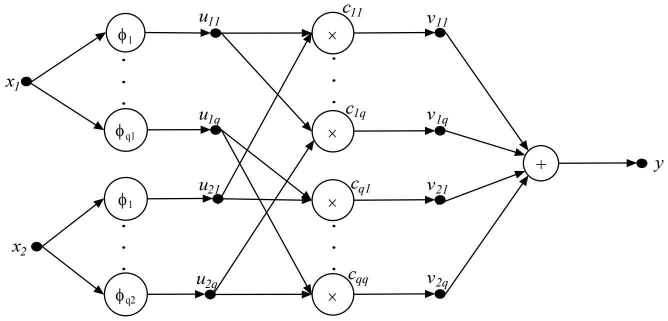


Fig. 1. Functional network general model for 2 inputs.

Functional networks consist of the following elements:

a) Several layers of storing units, which are represented in Figure 1 by small filled circles. These units are used for the storage of input, output and intermediate information. In the figure, there are four layers of storing units: the first layer contains two input units, x_1 and x_2; the second and third layers contain two sets of intermediate units, u_i and v_i; and finally, the third layer contains only one output unit y.

b) One or more layers of neurons, which are represented by open circles with the name of each of the functional units inside. Neurons are functional units, that represent a function and evaluate a set of input values in order to return a set of output values to the next layer of storing units. The functional network on Figure 1 has three layers of functional units. The first layer has q_1+q_2 neurons, and the second layer has q_1q_2 neurons. $c_{11}\ldots\ c_{q1q2}$, are some of the model parameters, $\Phi_S=\{\phi_{rs}(X_s),\ r_s=1,2,\ldots,q_s\}$ are families of linearly independent unknown functions, and q_s is the number of elements in Φ_S, where $s = 1,2,\ldots,k$. The general functional network model in Figure 1 can be looked at as a functional network representing the following semi-parametric regression model:

$$f(y)=\sum_{r_1=1}^{q_1}\cdots\sum_{r_k=1}^{q_k}c_{r_1\ldots r_k}\phi_{r_1}(x_1)\ldots\phi_{r_k}(x_k)\ . \quad \mathbf{(3)}$$

where f is the identity function.

a) A set of directed links. The computing units are connected to the storing units by directed arrows. The arrows indicate the direction of the flow of information. Once the input values are given, the output is determined by a function. For example, the outputs u_{iq} of ϕ_{iq} are the inputs to the summing units and v_{iq} are their outputs. The general functional network in the figure does not have any converging arrows, but if it did, this indicates that the neurons from which they emanate must produce identical outputs. This is an important feature of functional networks that is not available for neural networks. This converging arrows represent constraints which can arise from physical and/or theoretical characteristics of the problem under consideration.

3.2 Specification of the topology of the functional network

The specification of the initial topology for a functional network is based on the features of the problem we are facing [2]. Usually knowledge about the problem can be used in order to develop a network structure, although on the absence of this knowledge always the general model can be used.

From an analysis of our problem it can be stated that the value of the highest daily load peak seems to be affected by:

a. The temperature *t* due to the effect of heating systems. In this case, as shown in **Fig. 2**, it has been observed a linear relation between the mean daily temperature and the maximum daily load.

b. The daylight hours, which also depend on the time of the year. Analysing the evolution of the daily load along a year, it can be observed that two peaks are produced during the day, as shown in **Fig. 3**. The first one coincides with the beginning of the day (around 7-8 a.m.) and the second one with the end of the day (around 8-9 p.m) and can be interpreted as been caused, in part, by the need of artificial light. During the months with shorter days these peaks, specially the second one, will coincide with the working hours, therefore incrementing the load demand. This can be observed by comparing **Fig. 3a** and **Fig. 3b**. Also, a sinusoidal pattern in the behaviour of load demands can be observed when this variable is plotted against time (see **Fig. 4**)

c. Also, when analysing the maximum daily loads for every day of the week some differences are observed (see **Fig. 5**), related to the working days. In this sense, in general, there is no significant differences among non-week-end days, except for holydays, that always suppose a decrement on the load. On the other extreme are Sundays where the load is lesser and presents the same behaviour as holydays. Saturdays are in the middle of non-week-end and Sundays.

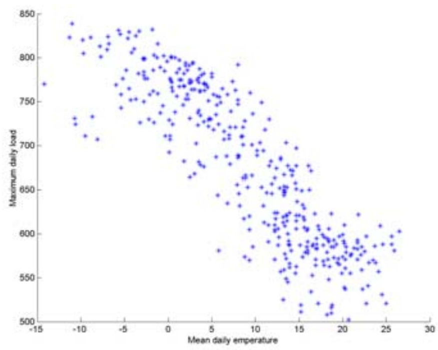
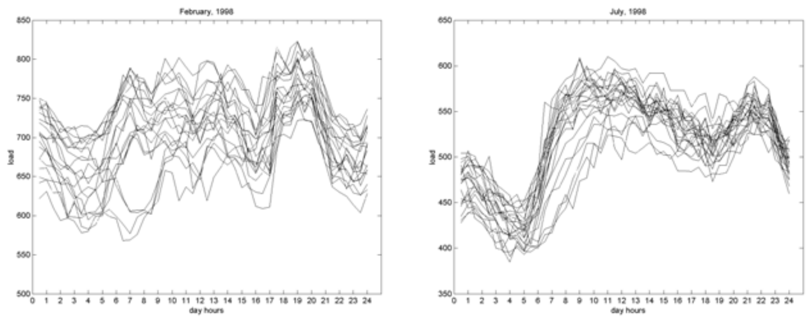


Fig. 2. Mean daily temperature versus maximum daily load for year 1998.



a) b)

Fig. 3. Half an hour daily electrical loads for February and July, 1998.

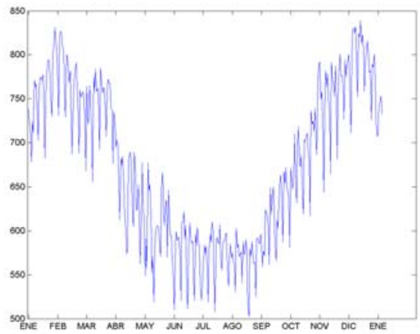


Fig. 4. Maximum daily load for 1998.

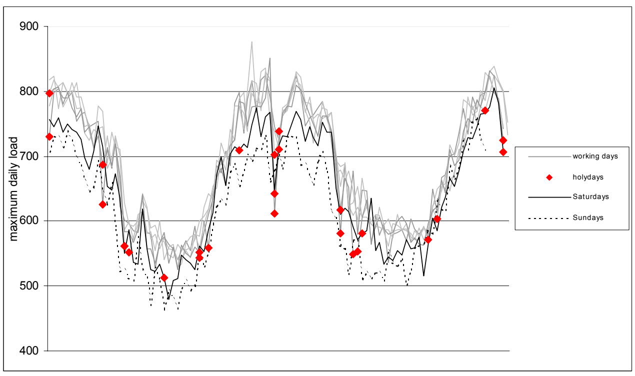


Fig. 5. Maximum daily loads by day of the week (year 1998)

After analysing the available data explaining the problem we chose, as an starting point, an additive model with the following characteristics.

To introduce the temperature terms into our model a one-degree polynomial family of functions was used, given the linear relationship with the maximum load. As inputs to these terms we will use n_1 means of k temperatures corresponding to the $n_1 \cdot k$ days immediately after and before the day we are predicting, using the available data for one and two years before this day.

To simulate the sinusoidal periodicity of the load we initially introduced terms from the Fourier family $sin(x)+ cos(x)+ sin(2x)+ cos(x)+\ldots+ sin(nx)+ cos(nx)$, where the period is $x= 2\cdot\pi/365$.

Finally, the polynomial family of functions was also used to introduce in the model terms regarding the history of the maximum daily loads. Thus, our initial model was stated as:

$$f(y) = c_0 + \sum_{n=1}^{n_1} c_{1n}\left(\frac{\sum_{j=i+(n-1)k}^{i+nk-1} t_{j_{m-1}}}{k}\right) + \sum_{n=1}^{n_1} c_{2n}\left(\frac{\sum_{j=i-(n-1)k-1}^{i-nk} t_{j_{m-1}}}{k}\right) + \sum_{n=1}^{n_1} c_{3n}\left(\frac{\sum_{j=i+(n-1)k}^{i+nk-1} t_{j_{m-2}}}{k}\right) +$$
$$+ \sum_{n=1}^{n_1} c_{4n}\left(\frac{\sum_{j=i-(n-1)k-1}^{i-nk} t_{j_{m-2}}}{k}\right) + \sum_{n=1}^{n_2} c_{5n} \sin\left(n \cdot \mathrm{mod}\left(\frac{i}{365}\right)\cdot p\right) + c_{6n} \cos\left(n \cdot \mathrm{mod}\left(\frac{i}{365}\right)\cdot p\right) +$$
$$\sum_{q=1}^{Q}\left(\sum_{n=1}^{n_3} c_{7qn}\left(\frac{\sum_{j=i+(n-1)k}^{i+nk-1} l_{j_{m-1}}}{k}\right)^q + \sum_{n=1}^{n_3} c_{8qn}\left(\frac{\sum_{j=i-(n-1)k-1}^{i-nk} l_{j_{m-1}}}{k}\right)^q\right) \quad \textbf{(4)}$$

where the indices i and m are, respectively, the day and year of the prediction, t are the mean daily temperatures, c are the maximum daily loads, and the period factor of the Fourier family functions is $p=2\pi/365$, trying to reflect its effect along the year.

Based on this initial model we developed three separated models: a first one for Sundays and holydays, a second one for Saturdays, and finally a third one for the rest of the days. These models only use the load variables l_j corresponding to the type of day each model is developed for. For example, in the model for Sundays l_{j-1} refers to the load of the Sunday/holyday before the i-th previous Sunday/holyday. However, all the temperature variables t_j are used in any model and, thus, in the model for Sundays t_{j-1} refers to the temperature of the day before i-th Sunday/holyday.

3 Results

3.1 Training and testing data

We built the training and test sets in order to have available for each day the temperature for that day one year and two years before, and the maximum daily load for that day one year before, as these were the available data. Also, this data were separated in three sets: one containing only Sundays/holydays, another containing only Saturdays and the final one containing the rest of the days. Each set was used to develop each of the corresponding models by making predictions for year 1998. For each model we employed 75% of data for training and the remaining for testing.

The load data was divided by a factor of 1000 to avoid problems during training.

3.2 The model for maximum daily electrical load prediction.

Several variants from the general model were trained in order to determine the optimal one whose expression, for the case of non-week-end, days is shown in Eq. (5), where n_3 is 3 for the non-week-end model and 1 for the rest of the models. From this expression, we can observe that only the cosines are used, which has sense if we look at maximal daily load curve in **Fig. 4**. Also, a second degree was determined as the optimal one for the polynomial containing the load terms. The coefficients' values are given in **Table 1**, and the results obtained are resumed in **Table 2**.

$$f(y) = c_0 + \sum_{n=1}^{3} c_{1n} \left(\frac{\sum_{j=i+3(n-1)}^{i+3n-1} t_{j_{m-1}}}{3} \right) + \sum_{n=1}^{3} c_{2n} \left(\frac{\sum_{j=i-3n+2}^{i-3n} t_{j_{m-1}}}{3} \right) + \sum_{n=1}^{3} c_{3n} \left(\frac{\sum_{j=i+3(n-1)}^{i+3n-1} t_{j_{m-2}}}{3} \right) +$$
$$+ \sum_{n=1}^{3} c_{4n} \left(\frac{\sum_{j=i-3n+2}^{i-3n} t_{j_{m-2}}}{3} \right) + \sum_{n=1}^{3} c_{5n} \cos\left(n \cdot \mathrm{mod}\left(\frac{i}{365} \right) \cdot p \right) +$$
$$+ \sum_{q=1}^{2} \left(\sum_{n=1}^{n3} c_{6qn} \left(\frac{\sum_{j=i+3(n-1)}^{i+3n-1} l_{j_{m-1}}}{k} \right)^{q} + \sum_{n=1}^{n3} c_{7qn} \left(\frac{\sum_{j=i-3n+2}^{i-3n} l_{j_{m-1}}}{3} \right)^{q} \right) \quad \textbf{(5)}$$

Table 1. Model's Coefficients. Model 1= model for non-week-end days, Model 2= model for Saturdays, Model 3= model for Sundays/holydays

Coefficients	Model 1	Model 2	Model 3
c_0	$4.6293\cdot10^{-3}$	$6.3174\cdot10^{-3}$	$6.048599\cdot10^{-1}$
c_{11}	$1.3902\cdot10^{-3}$	$-5.1691\cdot10^{-3}$	$1.1883\cdot10^{-3}$
c_{12}	$6.782\cdot10^{-4}$	$3.9969\cdot10^{-3}$	$-1.6019\cdot10^{-3}$
c_{13}	$-8.023\cdot10^{-4}$	$-6.82\cdot10^{-5}$	$4.4608\cdot10^{-3}$
c_{21}	$-3.221\cdot10^{-4}$	$2.9419\cdot10^{-3}$	$1.9189\cdot10^{-3}$
c_{22}	$-1.0296\cdot10^{-3}$	$1.9966\cdot10^{-3}$	$-4.070\cdot10^{-4}$
c_{23}	$^{-}2.510\cdot10^{-4}$	$-3.1692\cdot10^{-3}$	$-2.6917\cdot10^{-3}$
c_{31}	$3.086\cdot10^{-4}$	$-3.8198\cdot10^{-3}$	$-3.6880\cdot10^{-3}$
c_{32}	$3.938\cdot10^{-4}$	$2.2317\cdot10^{-3}$	$1.0696\cdot10^{-3}$
c_{33}	$-5.137\cdot10^{-4}$	$-3.6825\cdot10^{-3}$	$-2.5351\cdot10^{-3}$
c_{41}	$4.527\cdot10^{-4}$	$1.8025\cdot10^{-3}$	$1.2359\cdot10^{-3}$
c_{42}	$-2.0576\cdot10^{-3}$	$-9.394\cdot10^{-4}$	$2.4456\cdot10^{-3}$
c_{43}	$9.504\cdot10^{-4}$	$-1.2035\cdot10^{-3}$	$-2.2345\cdot10^{-3}$
c_{51}	$-4.83041\cdot10^{-2}$	$2.76978\cdot10^{-2}$	$7.10071\cdot10^{-2}$
c_{52}	$-3.647\cdot10^{-4}$	$-7.1970\cdot10^{-3}$	$1.567692\cdot10^{-1}$
c_{53}	$-2.5302\cdot10^{-3}$	$1.7135\cdot10^{-3}$	$-1.6122\cdot10^{-3}$
c_{611}	$-5.963\cdot10^{-4}$	$-1.707\cdot10^{-4}$	$-1.0382\cdot10^{-3}$
c_{612}	$-1.338\cdot10^{-4}$	$2.1930\cdot10^{-3}$	$1.1489\cdot10^{-3}$
c_{613}	$-1.189\cdot10^{-4}$	-	-
c_{621}	$-2.034\cdot10^{-4}$	-	-
c_{622}	$-1.0486\cdot10^{-3}$	-	-
c_{623}	$3.325\cdot10^{-4}$	-	-
c_{711}	$2.5367\cdot10^{-3}$	$2\cdot10^{-7}$	$9\cdot10^{-7}$
c_{712}	$7\cdot10^{-7}$	$-1.6\cdot10^{-6}$	$1\cdot10^{-6}$
c_{713}	$2\cdot10^{-7}$	-	-
c_{721}	$2\cdot10^{-7}$	-	-
c_{722}	$3\cdot10^{-7}$	-	-
c_{723}	$8\cdot10^{-7}$	-	-

Table 2. Model's error measures. Model 1= model for non-week-end days, Model 2= model for Saturdays, Model 3= model for Sundays/holydays

	Model 1	Model 2	Model 3
MAPE train error	1.921	1.857	1.973
Maximal train error	45.228	38.326	43.242
MAPE test error	3.430	6.396	4.241
Maximal test error	93.885	87.488	87.079

Finally, our predictions for January 1999 are given in **Table 3**.

Table 3. Maximum daily electrical load prediction for January, 1999.

Day	Predicted Load
1	750
2	792
3	752
4	766
5	769
6	721
7	778
8	774
9	783
10	728
11	758
12	763
13	762
14	762
15	762
16	771
17	717
18	775
19	777
20	787
21	799
22	817
23	748
24	689
25	815
26	815
27	812
28	808
29	805
30	773
31	692

5 Summary

Three models have been developed for the prediction of the maximum daily electrical load. To train and test the models the GAMS software was used. This is a computer program that allows to easily define, analyse and solve optimisation problems. The program has been executed on a MIPS R12000 Processor Chip Revision: 3.5 with 5120 Mbytes of memory size, for which the execution times were around 0.040 seconds. However, the simplicity of the obtained model makes it viable to implement it in a standard PC system, thus making possible the use of the prediction model at almost no additional cost with a high expected accuracy as indicated by the obtained MAPE errors.

References

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